Measurement noise

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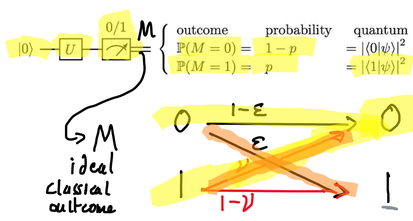
Introduction to quantum noise

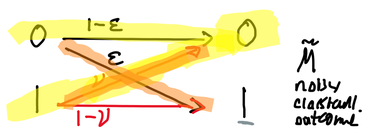
Measurement error

Qiskit Global Summer School on Quantum Machine Learning

Zlatko K. Minev



$\hat{M} = |1\rangle\langle 1| = \frac{1}{2}(\hat{I} - \hat{Z})$



M ideal classical outcome

$\tilde{M}$ noisy classical outcome

$$P_M = \begin{pmatrix} \mathbb{P}(M=0) \\ \mathbb{P}(M=1) \end{pmatrix} = \begin{pmatrix} 1-p \\ p \end{pmatrix} \qquad P_{\tilde{M}} = \begin{pmatrix} \mathbb{P}(\tilde{M}=0) \\ \mathbb{P}(\tilde{M}=1) \end{pmatrix} = \begin{pmatrix} 1-\tilde{p} \\ \tilde{p} \end{pmatrix}$$

$$\begin{cases} \mathbb{P}(\tilde{M}=0) = \mathbb{P}(\tilde{M}=0|M=0)\,\mathbb{P}(M=0) + \mathbb{P}(\tilde{M}=0|M=1)\,\mathbb{P}(M=1) \\ \mathbb{P}(\tilde{M}=1) = \mathbb{P}(\tilde{M}=1|M=0)\,\mathbb{P}(M=0) + \mathbb{P}(\tilde{M}=1|M=1)\,\mathbb{P}(M=1) = \varepsilon(1-p) + (1-\nu)\,p = \tilde{p} \end{cases}$$

(error)

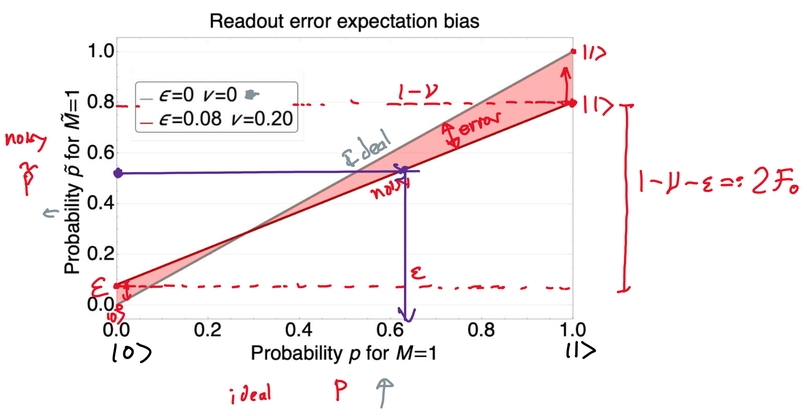
$P_{\tilde{M}} = A\,P_M$ ($\tilde{M}$: noisy, M: ideal; A: readout error matrix)

$$A = \begin{matrix} \\ \tilde{M}=0 \\ \tilde{M}=1 \end{matrix} \begin{pmatrix} \mathbb{P}(\tilde{M}=0|M=0) & \mathbb{P}(\tilde{M}=0|M=1) \\ \mathbb{P}(\tilde{M}=1|M=0) & \mathbb{P}(\tilde{M}=1|M=1) \end{pmatrix} \qquad (M=0,\ M=1)$$

$$\approx \begin{pmatrix} 1-\varepsilon & \nu \\ \varepsilon & 1-\nu \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ for an ideal measurement

$\sum_m A_{mn} = 1$ for any n — Stochastic matrix

$$\begin{aligned} \tilde{p} &= \varepsilon(1-p) + (1-\nu)p \\ &= \varepsilon - p\varepsilon + p - \nu p \\ &= p + \varepsilon - (\nu + \varepsilon)\,p \end{aligned}$$



Bonus section content:

Reconstruct A matrix

$|0\rangle$ — $(M=0)$ — A — $\tilde{M}$

$p=0$ $\qquad \tilde{p}=\varepsilon$

$|0\rangle$ — X — $|1\rangle$ — M — A — $\tilde{M}$ ← we have access to

$p=1$ $\qquad \tilde{p}=1-\nu$

Noise mitigation

we know A

measure $\tilde{p}$, $\tilde{P}_M$ noisy

find p, P_M ideal

$\dim A = 2^n \times 2^n$ $\quad n$ = #qubits

$$\tilde{P}_M = A\,P_M$$

$$P_M = A^{-1}\tilde{P}_M$$

$p =$

Assignment Fidelity

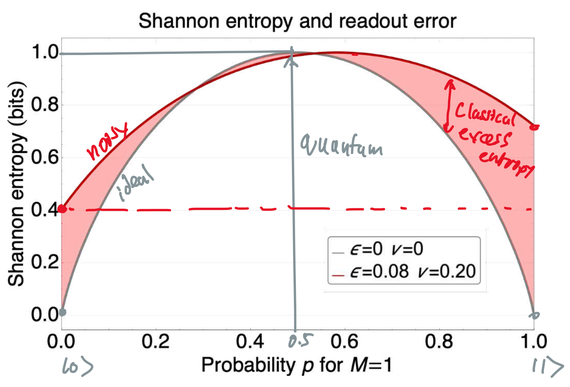
$$\begin{aligned} \mathcal{F}_0 &= 1 - \tfrac{1}{2}\left[\mathbb{P}(\tilde{M}=1|M=0) + \mathbb{P}(\tilde{M}=0|M=1)\right] \\ &= 1/d\ \mathrm{Tr}(A) \\ &= 1 - \tfrac{1}{2}(\nu+\varepsilon) \end{aligned}$$

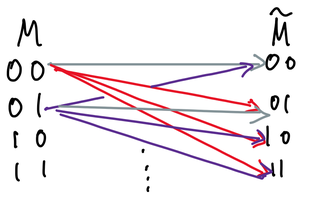
$d = 2^n$, n = #qubits

Shannon Entropy

$$\mathcal{H}(M) = \mathcal{H}(P_M) = -\sum_m P_m \log_2 P_m = -(1-p)\log_2(1-p) - p\log_2 p$$

Binary entropy



Larger system

n qubits $\qquad \dim A = 2^n \times 2^n$